Chapter 6: Frequency Domain Anaysis

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Outline

1. Introduction of Frequency Response
2. Frequency Response of the Typical Elements
3. **Nyquist and Bode Diagram of Open-loop System**
4. Nyquist-criterion
5. Frequency Response Based System Analysis
6. Frequency Response of Closed-loop Systems

Concept

Graphical method

Analysis

6.6.1 Dynamic performance analysis based on open-loop frequency response

1. Second-order time-domain response

Open-loop frequency response

$$A(\omega)=\frac{\omega_n^2}{\sqrt{\omega^4+(2\xi\omega_n\omega)^2}} \qquad \varphi(\omega)=-90^{\circ}-\mathrm{arctg}\frac{\omega}{2\xi\omega_n}$$

For $\omega=\omega_c$, we have $A(\omega)=1$, and then further get

$$\frac{\omega_c}{\omega_n}=\sqrt{\sqrt{4\xi^4+1}-2\xi^2}$$

Phase margin

$$\gamma = 180^{o} + \varphi(\omega_c) = \text{arctg}\left[2\xi\left(\frac{1}{\sqrt{4\xi^4+1}-2\xi^2}\right)^{1/2}\right]$$

The relationship between the phase margin and the damping ratio can be approximated as

Damping ratio of the closed-loop system ← $\xi = 0.01\gamma$ → **Measured form Bode diagram of open-loop system**

Overshoot

$$M_p = e^{-\frac{\xi\pi}{\sqrt{1-\xi^2}}} \times 100\%$$

Rule: For second-order systems, the smaller γ,the smaller ξ and the bigger M_p. The Bigger γ, the bigger ξ and smaller M_p.

For engineering application $30^{o} \le \gamma \le 60^{o}$

Settling time t_s

$$t_s\omega_c \approx \frac{3\sqrt{\sqrt{1+4\xi^4}-2\xi^2}}{\xi}$$

Rule: Given ξ, the bigger ω_c, the smaller t_s. The system response is faster.

2. Relationship between time-domain and frequency-domain performance for higher-order systems

- **The overshoot M_p increases when phase margin γ decreases. Settling time t_s is also increased for smaller γ, but it is decreased with bigger ω_c.**

5.6.2 Dynamic performance analysis based on closed-loop frequency response

1、Open-loop and closed-loop frequency responses

- **Given the unity feedback system with open-loop transfer function *G*(*s*)，the closed-loop frequency response is given by**

$$\Phi(j\omega) = \frac{C(j\omega)}{R(j\omega)} = \frac{G(j\omega)}{1+G(j\omega)}$$

- **Lower frequency part:**

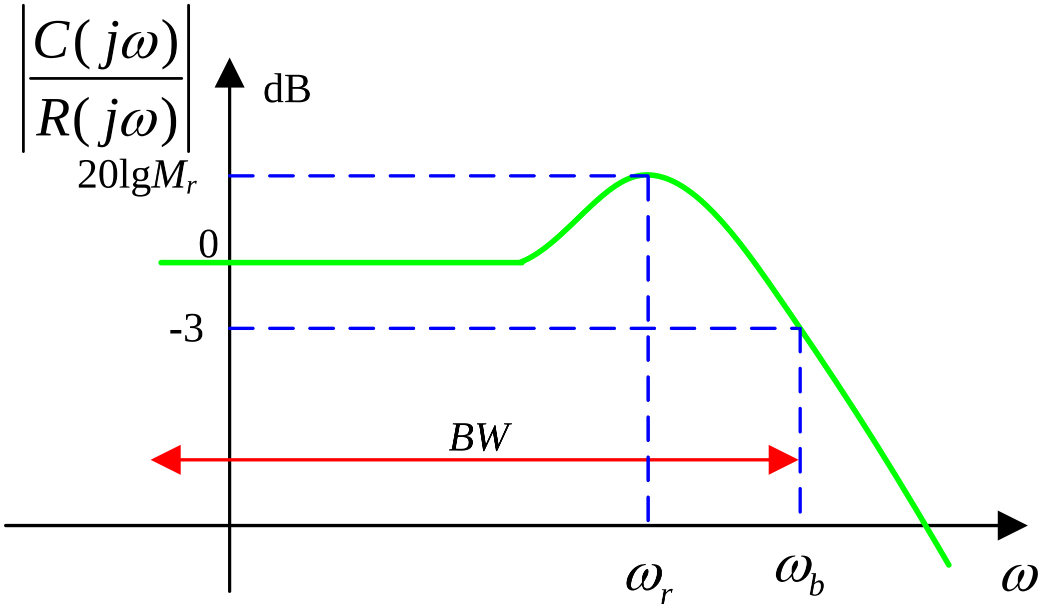
$$|\Phi(j\omega)| = \left|\frac{C(j\omega)}{R(j\omega)}\right| = \left|\frac{G(j\omega)}{1+G(j\omega)}\right| \approx 1$$

- **Higher frequency part**

$$|\Phi(j\omega)| = \left|\frac{C(j\omega)}{R(j\omega)}\right| = \left|\frac{G(j\omega)}{1+G(j\omega)}\right| \approx |G(j\omega)|$$

Normally, for minimum phase unity feedback system, the closed-loop output is almost equal to the lower-frequency input signal.

In the higher-frequency part, the closed-loop frequency response is similar to open-loop frequency response.



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2. Frequency-domain performance analysis for second-order closed system

Closed-loop frequency response

$$\frac{C(j\omega)}{R(j\omega)} = \frac{1}{(1-\frac{\omega^2}{\omega_n{}^2}) + j2\zeta\frac{\omega}{\omega_n}} = M(\omega)e^{j\varphi(\omega)}$$

(1) Resonant frequency

$$\omega_r = \omega_n\sqrt{1-2\zeta^2}$$

The frequency at which the maximum value of $\left|\frac{C(j\omega)}{R(j\omega)}\right|$ occurs is referred to as the resonant frequency ω_r.

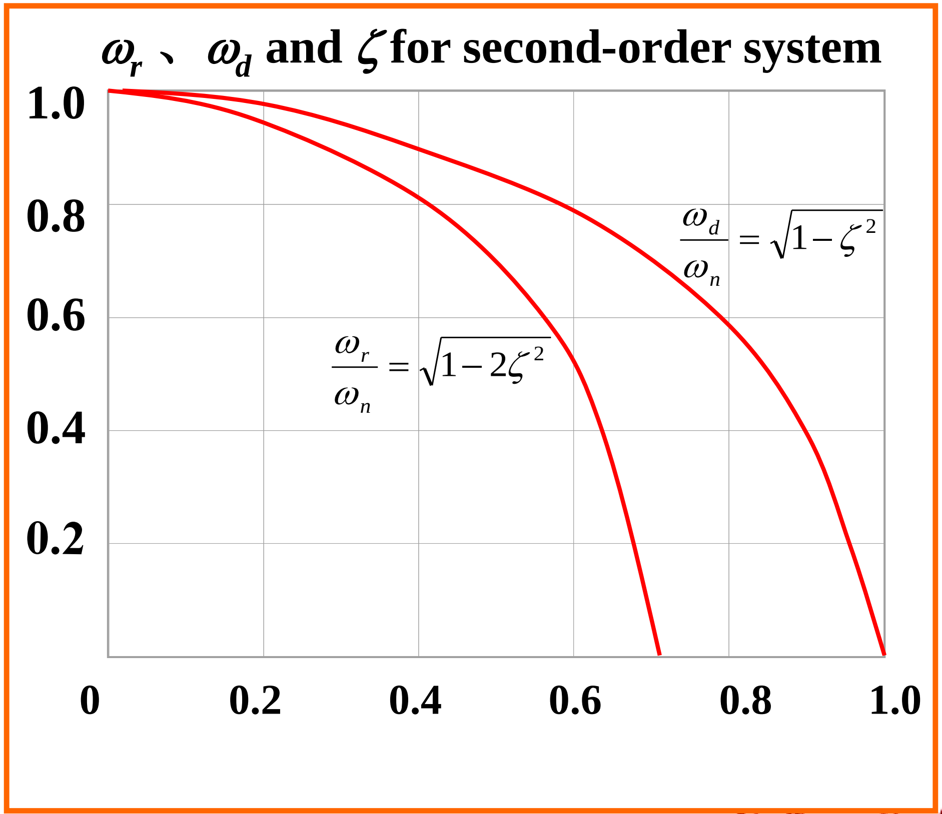
(2) Maximum value (Resonant peak)

$$M_r = M(\omega_r) = \frac{1}{2\zeta\sqrt{1-\zeta^2}}$$

The maximum value *Mr* is related to the overshoot of step response of the system.

The relative stability is better with the condition of smaller *Mr* and bigger ζ.

Normally, the ideal resonant peak is between 1.1 and 1.4. The corresponding ζ is in the region of 0.4<ζ<0.7.



(3) Bandwidth ω_b

The passband, or bandwidth, of the frequency response is defined as the range of frequencies from 0 to the frequency ω_b where the magnitude is $\frac{1}{\sqrt{2}}$ times of the value at $\omega=0$（Log-magnitude response decreases to 3dB less than the value of $\omega=0$）

$$\omega_b = \omega_n \sqrt{1 - 2\zeta^2 + \sqrt{2 - 4\zeta^2 + 4\zeta^4}}$$

For $\omega > \omega_b$, $20\lg|\Phi(j\omega)| < 20\lg(\frac{1}{\sqrt{2}} \cdot |\Phi(j0)|) = 20\lg|\Phi(j0)| - 3$

- **For open-loop system with $v \geq 1$, since** $20\lg|\Phi(j0)| = 0$

  we have $20\lg|\Phi(j\omega)| < -3\text{db}$

- **The bandwidth reflects the filtering of noise. The noise is generally in a band of frequencies above the dominant frequency band of the true signal.**
- **The bandwidth also reflects the response speed. The bigger the bandwidth is, the faster the response goes.**

Example 6.16: Given unity-feedback control system with Type I open-loop transfer function

$$G_0(s) = \frac{5}{s(s+1)(s+4)} = \frac{5}{s^3 + 5s^2 + 4s}$$

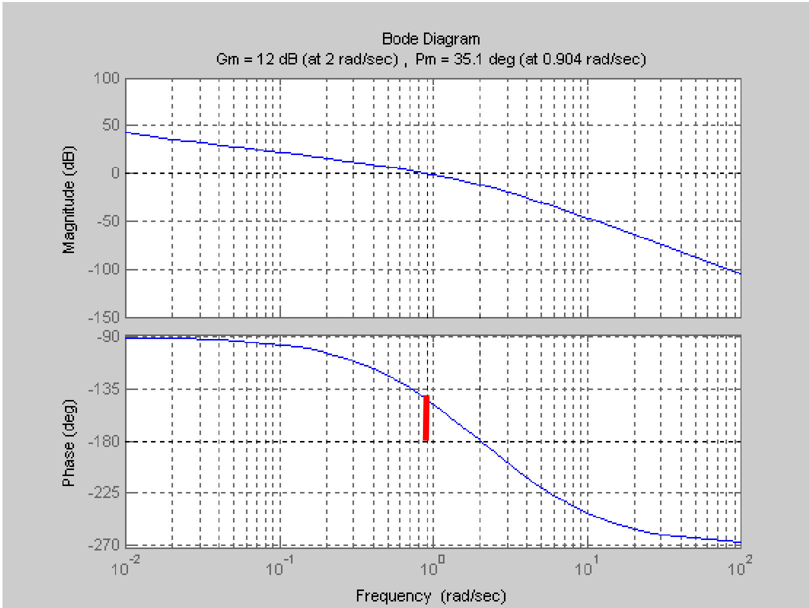
Now add one zero and one pole to the system, and the compensation is

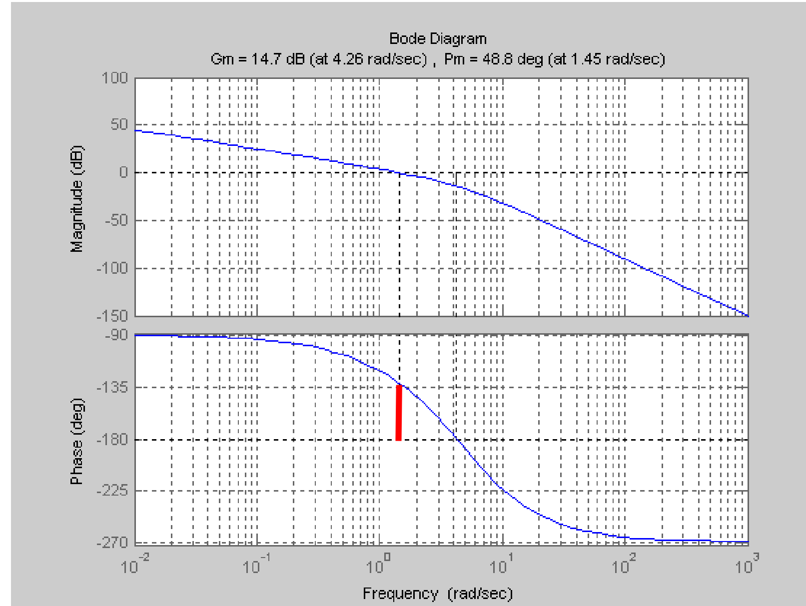
$$G_c(s) = \frac{5.94(s+1.2)}{(s+4.95)}$$

Please analyze the frequency response for two cases. (before and after adding the extra zero and pole)

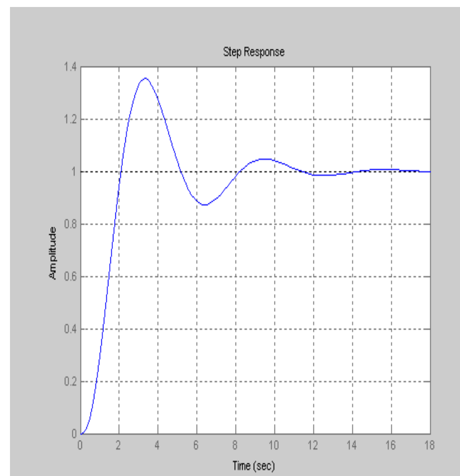
Solution: The transfer function with the compensation is given by

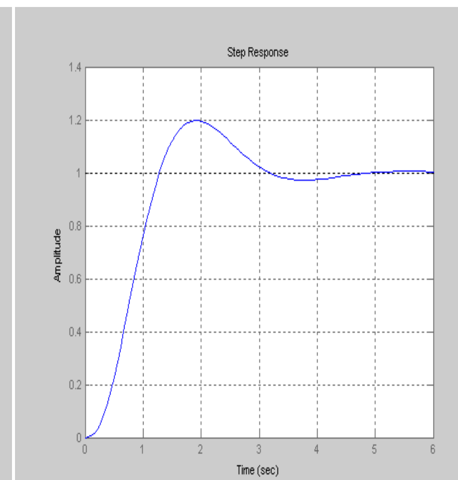
$$G(s) = \frac{29.7(s+1.2)}{s(s+1)(s+4)(s+4.95)}$$





- **It is observed from the figures that the phase margin and gain margin are both increased after adding the extra zero and pole.**
- **The more phase margin, the smaller overshoot.**





Summary of Chapter 6

Key points:

- Computing of frequency response
- Sketching of Nyquist diagram and Bode diagram
  - Nyquist diagram for open-loop system with integration elements
  - Asymptotes for Log-magnitude diagram
- Frequency domain stability criterion
  - Nyquist stability criterion（especially for systems with integration elements）
  - Stability criterion based on Bode diagram
- Relative stability

  Phase margin and gain margin (definition and graphs)